Synopsis

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A nonconforming least-squares method is proposed for the solution of elliptic boundary value problems over smooth domains. The least-squares approach is based on minimization of a functional which is the sum of squares of the L^2 norm of the residuala in the partial differential equation, the sum of residuals in the boundary conditions in fractional sobolev norms and continuity is enforced by adding a term which measures the sum of the squares of the jump in the function and its derivatives in fractional sobolev norms across interdomain boundaries. To incorporate periodicity of the functional identical values of the function are specified on the opposite sides of the boundary by adding a terms which measures the sum of the squares of the jump in the function and its derivatives in fractional sobolev norms.

The problem of homogenization of heterogeneous media is taken as a specific boundary value problem. The reason for this choice is two fold. The first reason is that homogenization is a powerful tool for multiscale analysis of heterogeneous media with rapidly oscillating materials parameters. In this study a popular engineering implementation is discussed, in which the solution is decomposed into a micro and a periodic micro part. The second reason is that heterogeneous media naturally results in non-simplistic domains. These domains are complicated and test the numerical methods well.

The spectral element functions are nonconforming on each quadrilateral. The elements are mapped onto the square S = (-1, 1) x (-1, 1) and the element function is represented as a sum of tensor products of polynomials of degree W in $\xi\, and\, \eta$, the transformed variables. A stability estimate for the functional which is being minimized. The normal equations for the least-squares problem are solved using preconditioned conjugate and storage of the mass and stiffness matrices, which are required for computation of the residuals in the normal equations.

We first above a much smaller system of equations corresponding to the schur complement of the sub vector of the common values, consisting of the values of the spectral element function at only one point of the domain. The schur complement matrix can be computed accurately since its dimension is small. This method turns out to be computationally more efficient than finite element methods since for solving the linear system associated with the discrete problem, it is very difficult to use direct method because the periodicity conditions disturb the band structure of the matrix. Moreover, complex techniques have to be preconditioned iterative solvers.

The well-posed ness of the transmission boundary value problem for the elasticity equation is proved, and the stability estimate is derived for the numerical scheme applied to the transmission problems.

For problems with no smooth inclusions, we used a geometric mesh at the corners. In a neighborhood of the corners, modified polar coordinates $(\tau_\kappa, \theta_\kappa)$ are used, where $\tau_\kappa = In\,\tau_\kappa$ and $(\tau_\kappa, \theta_\kappa)$ are polar coordinates with origin at the vertex A_k. Away form the sectoral neighborhood of the corner, a global coordinate system is used consisting of (x_1, x_2) coordinates elsewhere in the domain.

The spectral element functions are nonconforming and the approximation in the corner most elements is represented by a constant. The sub vector consisting of the common boundary values of the schur complement is the values of the function at the vertices of the domain only, so an exact approximation to the schur compliment can be computed since th e dimension the matrix is small.

The stability estimates are used to obtain a novel preconditioner for the solution of the minimization problem. Using this preconditioner, the condition of the system can be shown to be o $((\text{In } W)^2)$. Moreover, the preconditioner matrix is a block diagonal matrix such that each diagonal block corresponds to a different element, which can be easily inverted. The method requires o (W In W) iterations of PCGM to obtain the solution to exponential accuracy.

As a specific example, heterogeneous media with perforations and with crack inclusions are considered. Effective or homogenized properties are computed for micro-domain with several perforated inclusions, and for a micro-domain with an embedded crack. Results for domains with different materials, with a smooth inclusion are provided. This method can be extended to domains with different materials with nonsmooth inclusions, which we intend to do in future work.